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Linear Estimation Using I_lman Filter and Irq Implementation

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Abstract—*This paper reviews the importance of Kalman Filter for the process of Linear Filtering. Kalman Filtering is an algorithmic approach that solves the stabilization problems recursively to generate a statistically optimal estimate of the system. The problem associated with estimation is discussed first. This filter finds its application in the processes where the state of the system is to be tracked every time new measurements of the system position are taken. Some of the typical examples are quad copters, two wheel balancing system, velocity control of a satellite and navigation systems.*

1. INTRODUCTION

Estimation is the process of obtaining processed information that is to be used for carrying out a specific task. The processed information is obtained by applying some mathematical function on the input obtained from various sources along with previously stored data. Estimation is required because the input may be inconsistent- incomplete, uncertain or unstable. However the processed information obtained is usable because it is obtained from the best sources available.

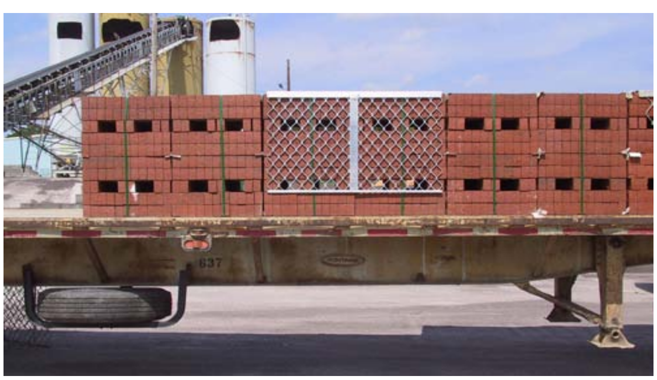

Fig. 1: The exact number of bricks in this truck cannot be determined by looking at it. The amount can be estimated by presuming that the portion of the truck that cannot be seen contains an amount equivalent to the amount contained in the same volume for portion that can be seen.

An estimator is simply a mathematical function of the given data. The function can be a linear or a non linear function; it can be a biased or an unbiased function. In this paper, we intend to look for an estimate that is linear as well as unbiased.

Currently we are working to implement this filtering algorithm in a project 'Two wheel balancing robot'. In order to obtain the raw values of the system orientation, we are using a sensor 'MPU6050' consisting of an accelerometer, gyroscope and a Digital Motion Processor(DMP) unit employing Kalman Filter. The filtered output from Kalman Filter wil be used as a feedback to our system.

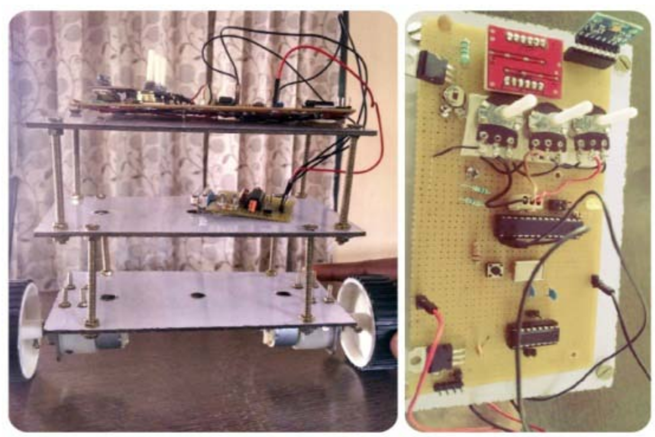

Fig. 2: Project under implementation

Kalman Filter which is named after Hungarian born electrical engineer, *Rudolf Emil Kalman* is also known as Linear Quadratic Estimator (LQE). It solves the least square estimation problems recursively going through data one by one. That is the filter finds the new best estimate for a given set of data once a new measurement is added by using both the new measurement, old estimate and some measure of confidence in the old estimate [1].

2. PROBLEM STATEMENT

Since the initial condition for the system is random, the system state cannot be measured directly. It has to be estimated optimally from measurements. So there is a need to apply an estimator block that inputs 'observed measurements' and previously recorded information to generate 'new best estimate' for the system.

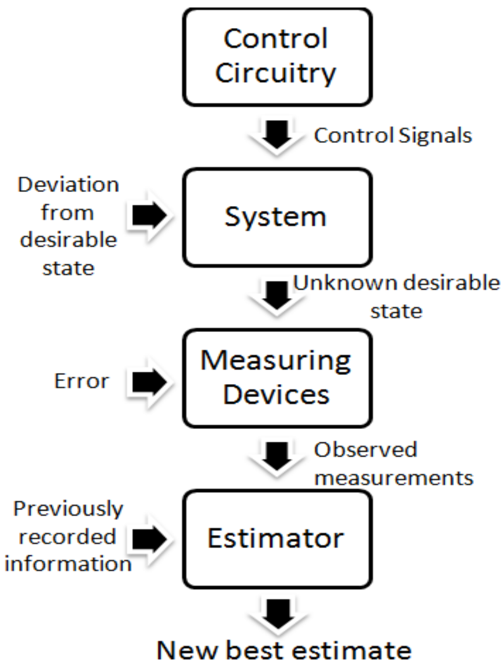

Fig. 3: Block Diagram for the Process

A linear system model is justifiable for a number of reasons. Often such a model is adequate for the purpose at hand, and when nonlinearities do exist, the typical engineering approach is to linearize about some nominal point of trajectory, achieving a perturbation model or error model. Linear systems are desirable in that they are more easily manipulated with engineering tools, and linear system (or differential equation) theory is much more complete and practical than nonlinear. The fact is that there are means of extending the Kalman filter concept to some nonlinear applications or developing nonlinear filters directly, but these are considered only if linear models prove inadequate [2].

3. THEORY OF ESTIMATION

Let us consider a quantity Y that is unknown. Two independent pieces of information Y_1 and Y_2 are available about the quantity Y.

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Mean of $Y_1 = E(Y_1) = m$

Variance of $Y_1 = Var(Y_1) = \sigma_1^2$

Mean of $Y_2 = E(Y_2) = m$

Variance of $Y_2 = Var(Y_2) = \sigma_2^2$

$\hat{Y}_{LS}$ = least square estimate of unknown Y

$\hat{Y}$ = any other estimate

Then according to Gauss- Markov theorem,

Covariance $(\hat{Y}_{LS}) \leq$ Covariance $(\hat{Y})$

Where $\hat{Y}_{LS}$ and $\hat{Y}$ are both linear and unbiased estimates.

Let us assume: $\hat{Y} = a_1 Y_1 + a_2 Y_2$

this is a linear estimate in Y_1 and Y_2

For any estimate to be unbiased the sum of coefficients should be 1.

i.e $a_1 + a_2 = 1$

$$Var(\hat{Y}) = a_1^2\, \sigma_1^2 + a_2^2\, \sigma_2^2 \quad \text{-----(1)}$$

It is required to find out the value of a_1 such that $Var(\hat{Y})$ is minimum.

$$\frac{\partial Var(\hat{Y})}{\partial a1} = 2\, a_1\, \sigma_1^2 + 2(1-a_1)(-1)(\sigma_2^2) = 0$$

$a_1 = \sigma_2^2 / (\sigma_1^2 + \sigma_2^2)$

$a_2 = 1 - a_1 = \sigma_1^2 / (\sigma_1^2 + \sigma_2^2)$

By substituting the values of a_1 and a_2 in equation1 and simplifying,

$Var(\hat{Y}) = \sigma_1^2\, \sigma_2^2 / (\sigma_1^2 + \sigma_2^2)$

By observation it can be viewed that:

$Var(\hat{Y}) < \sigma_1^2$

And $Var(\hat{Y}) < \sigma_2^2$

this means that: $Var(\hat{Y}) < \min\{ \sigma_1^2, \sigma_2^2 \}$

Hence $\hat{Y}$ is a better estimate as compared to the individual data sets Y_1 or Y_2 as variance of $\hat{Y}$ is the least[6,7].

Now let us assume that Y_1 is the background i.e. previously recorded information and Y_2 is the new observation.

We have two pieces of information and it has just been proved according to the theory of estimation that the linear unbiased estimate obtained from the combination of two or more information results is a better estimate. This is the reason, why Data Simulation is carried out. Kalman filter is based on this principle of estimation.

In a linear model for loss reserving, Gauss Markov prediction is the natural principle of prediction: It minimizes the mean squared error of prediction over the class of all unbiased linear predictors, and it provides exact formulas for predictors and their mean squared error of prediction. Another advantage of

Gauss Markov prediction is in the fact that the Gauss Markov predictor of a sum is just the sum of the Gauss Markov predictors of the single terms of that sum such that essentially only the most elementary quantities have to be predicted. [5]

4. KALMAN FILTERING PROCESS

The filter is actually a data processing algorithm. Despite the typical connotation of the filter as a "black box" containing electrical networks, the fact is that in most practical applications, the "filter" is just a computer program in a central processor. As such, it inherently incorporates discrete time measurement samples rather than continuous time inputs [2].

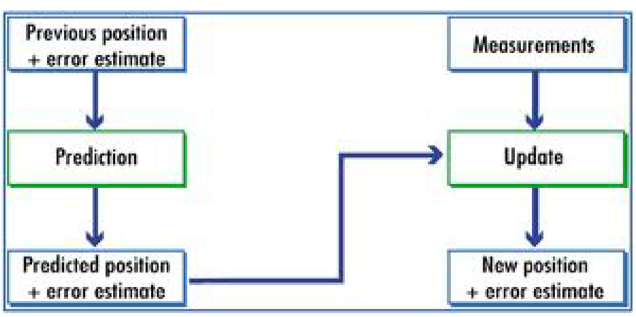

Fig. 4: Block diagram of Kalman filtering process [8]

For linear system and white Gaussian errors, Kalman filter is "best" estimate based on all previous measurements. It does not need to store all previous measurements as it reprocesses all data at each time step.

- Optimal estimate of position is: $\hat{y}(t_1) = z_1$
- Variance of error in estimate: $\sigma^2_x(t_1) = \sigma^2_{z1}$
- Measurement at t_2: Mean = z_2 and Variance = σ^2_{z2}

There is a need to correct the prediction due to measurement to get $\hat{y}(t_2)$. This is done by using Linear Estimation which gives corrected mean. Corrected mean is the new optimal estimate of position. New variance is smaller than either of the previous two variances [4].

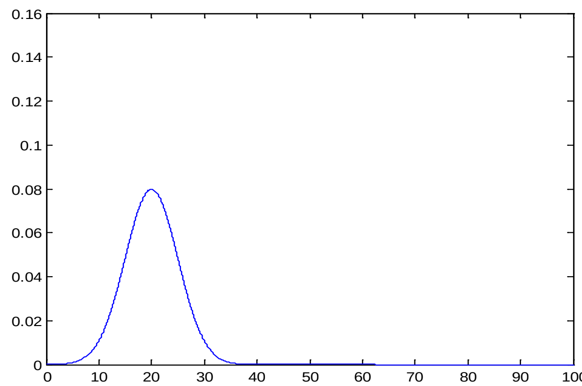

Fig. 5: Predicted position[2]

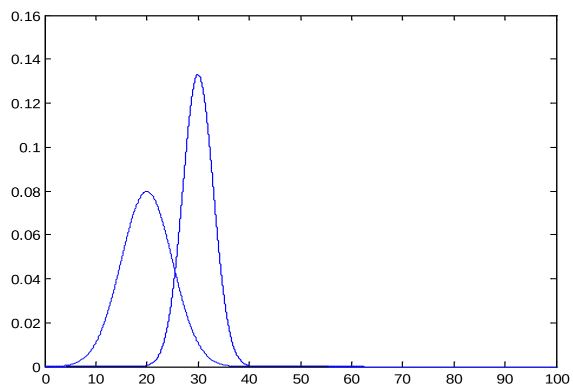

Fig. 6: Measured position[2]

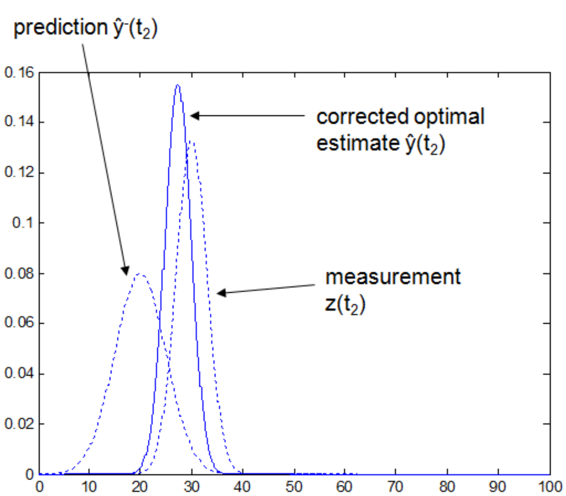

Fig. 7: Optimally estimated position [2]

The linear discrete Kalman filter can be used to estimate the state of a process for which information is constantly gathered or any problem where one wants to have a state estimate quickly, before all the information is processed. An example of the former is navigation, or weather prediction. In both of these cases we want ongoing solutions each time data is added. An example of an application where all the data is immediately available but not all of it is used right away is a quad copter or a two wheel balancing robot. In these cases, high speed estimation of the deviation of the assembly from the required coordinates is essential. The a priori state estimate given by the Kalman filter after only a few data points can be used to determine whether the position of the assembly should be updated or not [1,4].

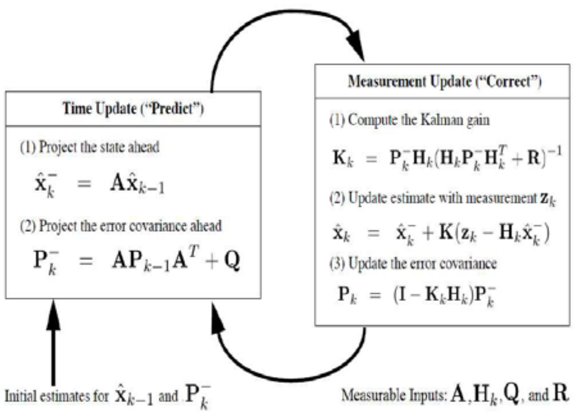

Fig. 8: Implementation diagram of Kalman Filter [3]

REFERENCES

[1] Introduction to the Kalman Filter and its Derivation; Brent Perreault; Concordia College, Moorhead, Minnesota; April 19, 2012 .
[2] Maybeck, Peter S., Stochastic Models, Estimation, and Control, Vol 1
[3] G. Welch and G. Bishop, An Introduction to the Kalman Filter, Citeseer, Vol. 7 (2006), pp. 1-16.
[4] R.E. Kalman, A new approach to linear filtering and prediction problems, Transactions of the ASME-Journal of Basic Engineering D, Vol. 82 (1960), pp. 35-45.
[5] Gauss Markov Loss Prediction in a Linear Model; Alexander Ludwig and Klaus D. Schmidt; Casualty Actuarial Society E-Forum, Fall 2010
[6] Department of Economics, Rutgers school of Arts and Sciences; URL: http://econweb.rutgers.edu/tsurumi/blue1.pdfs
[7] Gauss Markov theorem by Dr. Frank Wood; Robotics Research Group, Department of Engineering Science, University of Oxford; URL:http://www.robots.ox.ac.uk/~fwood/teaching/W4315_Fall2010/Lectures/gauss_markov_theorem/main.pdf
[8] Argos- worldwide tracking and environmental monitoring by satellite .
[9] URL:http://www.argos-system.org/manual/3-location/32_principle.htm